Exercise 70

A clothing business finds there is a linear relationship between the number of shirts, n, it can sell and the price, p, it can charge per shirt. In particular, historical data shows that 1,000 shirts can be sold at a price of \$30, while 3,000 shirts can be sold at a price of \$22. Find a linear equation in the form $p(n) = mn + b$ that gives the price p they can charge for n shirts.

Solution

The linear equation has the form,

$$p(n) = mn + b.$$

Based on the historical data, the two points on the line are $(1000, 30)$ and $(3000, 22)$. The first point says that when the input is 1000, the output is 30.

$$30 = m(1000) + b$$

The second point says that when the input is 3000, the output is 22.

$$22 = m(3000) + b$$

This is a system of two equations for two unknowns that can be solved for.

$$\begin{cases} 1000m + b = 30 \\ 3000m + b = 22 \end{cases}$$

Subtract the respective sides to eliminate b.

$$1000m - 3000m = 30 - 22 \quad \rightarrow \quad -2000m = 8 \quad \rightarrow \quad m = -\frac{2}{500}$$

Multiply both sides of the first equation by -3

$$\begin{cases} -3000m - 3b = -90 \\ 3000m + b = 22 \end{cases}$$

and then add the respective sides to eliminate m.

$$-3b + b = -90 + 22 \quad \rightarrow \quad -2b = -68 \quad \rightarrow \quad b = 34$$

Now that m and b are solved for, the line is known.

$$p(n) = -\frac{2}{500}n + 34$$